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Monte Carlo simulation of Ising model on directed Barabasi-Albert network

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Abstract:

The existence of spontaneous magnetization of Ising spins on directed Barabasi-Albert networks is investigated with seven neighbors, by using Monte Carlo simulations. In large systems we see the magnetization for different temperatures T to decay after a characteristic time $\tau(T)$, which is extrapolated to diverge at zero temperature.

Keywords: Monte Carlo simulations, Directed Barabasi-Albert networks, Magnetization, Fortran program

Introduction:

The Ising magnet is since decades a standard tool of computational physics [1]. We apply it here to scale-free networks [2], where previous simulations [3] indicated a Curie temperature increasing logarithmically with increasing system size N. In contrast to that work we use here directed [4] as opposed to undirected networks and then apply the standard Glauber kinetic Ising model [1] to the fixed network.

Directed Barabasi-Albert network:

Putting Ising spins onto the sites (vertices, nodes) of a network, we simulate our Ising magnetic model on directed Barabasi-Albert networks. The Barabasi-Albert network is grown such that the probability of a new site to be connected to one of the already existing sites is proportional to the number of previous connections to this already existing site: The rich get richer. In this way each new site selects exactly m old sites as neighbours.

Then each spin is influenced by the fixed number m of neighbours which it had selected when joining the network. It is not influenced by other spins which selected it as neighbour after it joined the network.

The Barabasi-Albert network is simulated by a Fortran program calculating the neighbours:

```
      parameter( nsites=500000,m=7,iseed=3, maxmax=20000,
     1 max=nsites+m, length=1+2*m*nsites+2*m*m , T=1.0)
      integer*8 ibm, iex
      dimension list(length), is(max), iex(2*m+1), neighb(max,m)
      ibm=iseed-1
      factor=(0.25d0/2147483648.0d0)/21474836484.0d0
      do 7 i=1,m
        do 7 nn=1,m
          neighb(i,nn)=nn
 7        list((i-1)*m+nn)=nn
      L=m*m
c     All m initial sites are connected
      do 1 i=m+1,max
        do 2 new=1,m
 4        ibm=ibm*16807
          j=1+(ibm*factor+0.5)*L
          if(j.le.0.or.j.gt.L) goto 4
          j=list(j)
          list(L+new)=j
          list(L+m+new)=i
 2        neighb(i,new)=j
 1      L=L+2*m
c     print *,ibm,neigh
c     end of network and neighbourhood construction
```

At each step, a new spin is added which builds m new connections `neighb`, randomly to already existing spins. The probability for an existing spin to be chosen as neighbour is proportional to the number of its neighbours, with the help of the Kertesz `list`.

Ising Magnet using Monte Carlo Simulations:

First we initialize a directed Barabasi-Albert network with m neighbours (all m initial spins are connected with each other and themselves), here $m = 2$ and 7. We put an Ising spins onto every site, with all spins up, because we test here for ferromagnetism. Then with the standard Glauber (heat bath) Monte Carlo algorithm spins we search for thermal equilibrium at positive temperature.

After putting all spins on the network, we go through the whole network and use the Monte Carlo step (MCS) on every spin; we say that we make

one MCS per spin at each time step. Each spin is influenced by its exactly m neighbours. We calculate the magnetization versus the number of time steps, with the same number of neighbours m and different temperatures T.

Initially we start with all spins up, a number of spins equal to 500,000, and time up to 20,000. Then we vary the temperatures and study $m = 2$ and 7 for nine samples (nine different random number sequences). The temperature is measured in units of the critical temperature of the square-lattice Ising model.

So we can draw a graph of magnetization versus time for different temperatures to see how the magnetization changes, Fig. 1.

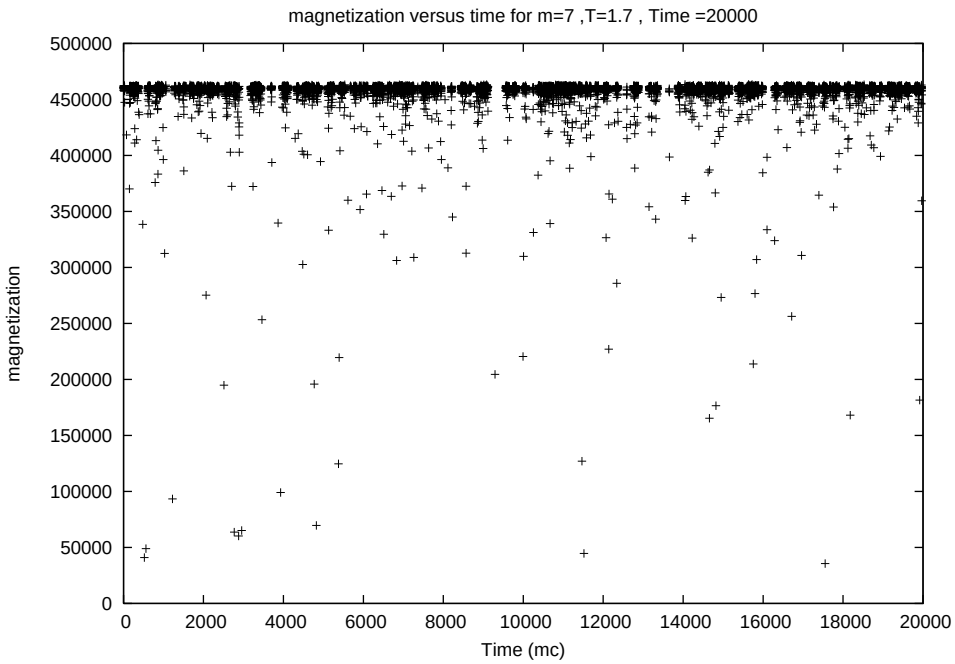

Figure 1: Magnetization versus MCS per spin, for $N = 500000$, time up to 20000, $m = 7$, for temperature $T = 1.7$.

We determine the time τ after which the magnetization has decayed to 3/4 of its initial value (here 375,000) for the first time, and then take the median value of our nine samples. So we get different values of τ_1 for different temperatures. Then we plot them double-logarithmically versus temperature in Fig.2 and as $1/\ln\tau$ versus temperature in Fig. 3.

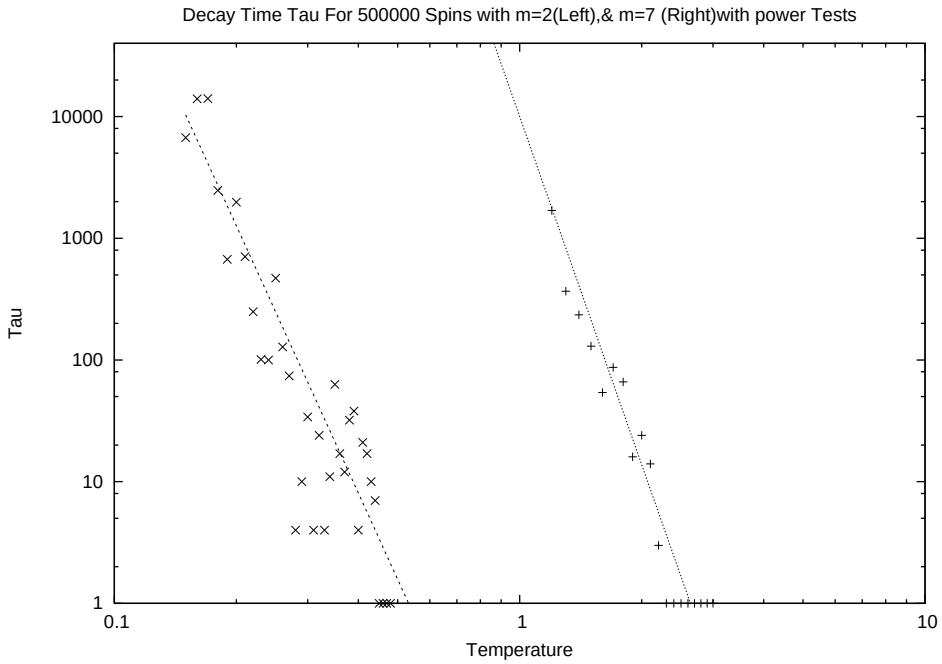

Figure 2: τ versus temperature for $N = 500000$, time up to 20000, $m = 2$ (left) and 7 (right). Each symbol is the median of nine samples. The straight lines have different slopes and thus correspond to power laws with different exponents.

Since the power-law test of Fig.2 gives two different exponents, we prefer the fits of Fig.3 to an Arrhenius law as given in the headline of the figure.

Conclusion:
We see that our results agree with the modified Arrhenius law:

$$1/\ln(\tau) = \text{const}(m) \cdot T + O(T^3)$$

meaning that for each positive temperature there is a finite relaxation time after which the initial magnetization decays towards zero: Similar to the one-dimensional Ising model there is no ferromagnetism on this directed Barabasi-Albert network.

References:

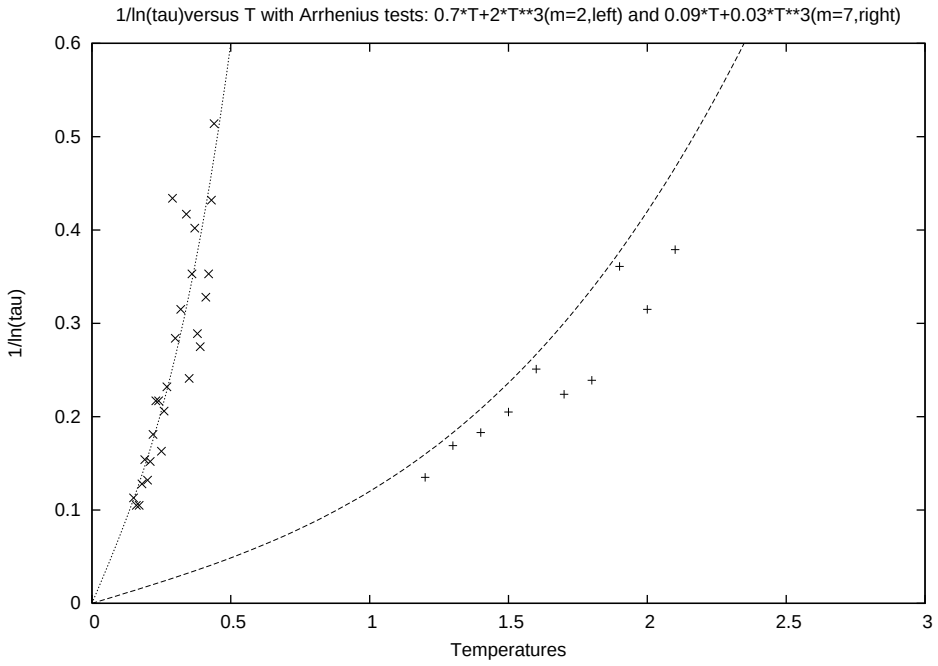

Figure 3: $1/\ln(\tau)$ versus temperature same data as in Fig. 2 The curves are parabolas correponding to an asymptotic Arrhenius law $\tau \propto \exp(\mathrm{const}(m)/T)$.

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